Discussion 2 Worksheet

Tangents, Area, Arclength

Date: 8/30/2021

MATH 53 Multivariable Calculus

1 Computing Tangents

Compute the slopes of the following curves at a point in time t. Find the points where the tangents are vertical and horizontal and compute the second derivative d^2y/dx^2 at the horizontal points.

(a) $x = \cos t, y = \sin t$

(b) $x = t^2 - 1$ $y = t^3 - t$

(c) $x = e^t - 1, y = \sin t$

(d) $x = e^t - t, y = \cos t$

2 Computing Areas

Using the appropriate formula, find the area in question.

(a) Use the parametric equations of an ellipse, $x = a\cos\theta$, $y = b\sin\theta$, $0 \le \theta \le 2\pi$, to find the area that it encloses.

(b) Find the area enclosed by the x-axis and the curve $x = t^3 + 1$, $y = 2t - t^2$.

(c) Find the area of the region enclosed by the astroid $x = a\cos^3\theta$, $y = a\sin^3\theta$.

3 Computing Arc Lengths

Using the appropriate formula, find the length of the curve.

(a) $x = 1 + 3t^2$, $y = 4 + 2t^3$, $0 \le t \le 1$.

(b) $x = e^t - t$, $y = 4e^{t/2}$, $0 \le t \le 2$.

(c) $x = e^t \cos t$, $y = e^t \sin t$, $0 \le t \le \pi$.

4 True/False

(a) T F The parametric representation of a curve is unique.

(b) T F When integrating, we can replace $\sin^2\theta$ with $(1 - \cos 2\theta)/2$.

(c) T F A (parametric) curve can only be described in either Cartesian coordinates $x = f(t), y = g(t)$ or in polar coordinates $r = f(\theta)$, but not both.

(d) T F $\sin(2t) = 2\sin t\cos t$

Note: These problems are taken from the worksheets for Math 53 in the Spring of 2021 with Prof. Stankova.